Teaching Notes

- Look for opportunities to get children to observe, describe, continue patterns and relationships, for example:

**Patterns**
20 – 5  Can you see a pattern in these subtractions?
21 – 6  Can you continue the pattern?
22 – 7  Why do they all produce the same answer?

**Commutativity**
10 × 5  Are these the same? Are they different?
5 × 10  How can we show the difference?
(This could be followed up with Sum Stories: 'Ten five-cent coins' as opposed to 'Five ten-cent coins')

- As follow up, or for evaluation, sometimes ask students to write down in a given time—say 2 minutes—as many calculations as they can which give a particular number.

- The use of calculators in this activity:
  - helps children experiment before going public
  - helps less confident children
  - allows more competent children to experiment with more complex expressions.

But make sure the mental aspect is stressed: use of calculators can lead to children producing long-winded expressions. A solution is to limit the number of operations allowed e.g. 'not more than three operations e.g. 4 × 2 + 3 × 1.'

- Don't be too bothered about use of brackets until years 6/7. The main thing is that children understand the convention used. For example 15 – 3 × 2 is perfectly acceptable as a record of how to produce 9, so long as children can verbalise the meaning e.g. 'take three times two, that is six, away from fifteen'. On the other hand some teachers of upper primary classes have remarked that this provides a great opportunity to discuss the usefulness of brackets in making clear what is meant.

- Avoid evaluating children's responses: don't say 'That's a great one, Fred' or 'Can someone give me a more interesting one?'.

Today's Number Is...

Objective: to develop confidence with numbers and operations and describe patterns and relationships

Materials: blackboard

Organisation: whole class

Procedure:
1 Teacher writes a number on the board (a number between 10 and 30 seems to work best for most classes). Students make up calculations for which the number is the answer.
2 As students suggest calculations, teacher writes them on the board in a structured way (see example below). Accept any answers. Put the onus on the class to tell you if any calculation is incorrect, thus involving all the children.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6 + 6</td>
<td>15 – 3</td>
<td>3 × 4</td>
<td>24 ÷ 2</td>
</tr>
<tr>
<td>4 + 8</td>
<td>20 – 8</td>
<td>6 × 2</td>
<td>12 + 1</td>
</tr>
<tr>
<td>0 + 12</td>
<td>112 – 100</td>
<td>1 × 12</td>
<td>1 × 2 × 2 × 3</td>
</tr>
</tbody>
</table>

3 After 8 or 10 responses, focus on particular columns or types of response. For example, point to first column:

Give me some more like this.
No more like this.
Give me some ways which use three numbers.
Some ways teachers have adapted this format:

- One year 7 class used calculators to explore more ways of making 8, for example they came up with:
  \[
  \sqrt{4} + 6 \\
  \sqrt{9} + 5 \\
  \sqrt{16} + 4
  \]

- A year 2 class used counters and rulers. They were given two minutes to write down their answers and were then asked to 'call out' their answers.

- A year 1 teacher has all the children on the mat for this activity as it focuses attention better, but children are free, and indeed encouraged, at any time to return to their desks to use counters to work something out.

- A year 4 teacher wrote some of the operations in words on the board, e.g. 'halve 36', 'triple 6'.

- A year 1 teacher takes the less able children while a teacher aide takes the more able children for the same activity. Each has the children round a table with a large sheet of paper. The two groups compete to fill the sheet. The more able children are encouraged to produce more original examples. The less able children were noticeably enjoying the activity: they laughed when they found that \( 6 + 0 \) produced the same result as \( 6 - 0 \), also when \( 1 + 2 + 1 + 2 \) was offered after \( 2 + 1 + 2 + 1 \). We should recognise such incidents as real examples of the power of mathematics to affect children of all abilities.

Extracts from lessons:

- A year 1 teacher started with the number 23. One child offered \( 100 - 87 \). To make this intelligible the teacher asked 10 children to stand up and extend all their fingers. They counted 100 in tens. Then the children removed 87 fingers, first in tens, then ones. They checked that only 13 fingers, not 23, remained. This shows how the teacher can take advantage of the activity to move into place value.

- Examples produced by the year one class for 23 (they had been using the activity for some months):
  \[
  (2 \times 10) + 3 \\
  4 \times 5 + 3 \\
  12 + 12 - 1 \\
  17 + 6 \\
  10 + 10 + 3 \\
  19 + 12 - 11 + 3 \\
  46 + 2 \\
  11 \text{ groups of } 2 + 1 \\
  5 + 2 + 2 + 1 + 2 + 3 + 1 + 1 + 2 + 1 + 3. \\
  50 - 23 \text{ (corrected by teacher)}
  \]

- Some responses to 20 by a less able year 4 group:
  - \( 10 - 5 + 15 \)
  - \( 22 - 2 \)
  - \( 110 - 90 \)
  - \( 34 - 14 \)
  - \( 100 - 80 \)
  - \( 30 - 10 \)
  - \( 1000 - 980 \)
  - \( 4 \times 5 \)
  - \( 6 \times 6 - 16 \)
  - \( 10 \times 2 \)
  - \( 5 \times 5 - 5 \)
  - \( 1 \times 20 \)
  - a dozen + 2 + 6 double 5 add 10 triple 5 add 5 20 + 20 - 20
  - half 40
  - 100 - 90 + 10 double 5 and double 5
  - half of 30 add 5

- In one class a child offered 20 - 6 to make 14. A little later another child offered 20 - 7. The teacher deliberately wrote these two next to each other on the board to see whether children would observe that they can't both be right. It may be that we need to give some deliberate opportunities of this kind to children, for example: 'Which of these three cannot give the same result as the other two?'

(a) \( 20 - 6 \)  (b) \( 3 \times 4 \)  (c) \( 128 + 79 \)

\[
\begin{align*}
21 - 7 & = 4 \times 3 \\
20 - 7 & = 5 \times 3 \\
& = 127 + 78 \\
& = 129 + 78
\end{align*}
\]
HOW DID YOU DO IT?

Objective: to use a variety of strategies for mental computation

Materials: blackboard

Organisation: whole class or whole class in groups of 2–4

Procedure:
1 Teacher gives a calculation orally or writes it on board horizontally.
2 Children calculate it mentally, but write nothing.
3 Children discuss their methods of solution in their groups or teacher directly asks three or four individuals for their method of calculation. For example:
   * Who can tell me how they worked that out?
   * Who else did it exactly the same way?
   * Who did it differently?
   * How did you do it?
4 Discuss variety of methods used. Stress the variety of methods used and the connections between the methods used and the original calculation rather than any decision about the ‘best’ method.

Comments on the activity:
- It is not easy at first to follow children’s explanation (though it becomes easier with practice). For example one child, explaining how she had calculated $37 + 15$, began ‘7 add 3 equals 10’. The teacher almost intervened, thinking that the child had misheard the question, since there was no ‘3’ in it. The child continued ‘$7 + 3$ equals 10, and 2 makes 12.’ It was then obvious that the child had used the very common strategy of ‘bridging ten’.
- It is important that you insist that all children listen quietly to the explanations, and that you make them feel that the explanations are important and interesting.
- It is hard, but necessary, to avoid jumping to conclusions too quickly about the child’s strategy, and to avoid putting words into the child’s mouth. Instead ask more questions:
   * What number did you begin with?
   * What did you do first? Why?
   * What did you do next?
- The emphasis should always be on strategy sharing, not on the difficulty of the problem.
- One year 6 teacher noted that at first the children were only concerned with the answer, but when they heard two or three different methods described they liked it and began to use the strategy which appealed most to them.

Some ways teachers have adapted this format:
- One teacher found that, for young children, drawing a number line on the board helped children to ‘see’ the explanations.
- If a child could not work out an answer the teacher asked: ‘Would you like someone to help you or would you like to do it yourself?’
- When a child produces an efficient strategy, one teacher focuses on this one strategy and gives more calculations for the whole class to practise using this strategy.
A year 2 teacher claps her hands, leaving pauses at (sometimes irregular) intervals, and asks the children: 'How many claps? How did you work it out?' Two examples were:

clapclapclap clapclapclap clapclapclapclapclap clapclapclapclapclap clapclapclapclapclap clapclapclap

This was a good activity for recognising patterns in counting. The children did not find it easy. Responses for the first of the two examples varied from 5 to 12, and for the second from 9 to 15.

A year 6 teacher incorporated this activity into Sum Stories (see page 36).

Extracts from lessons:

- Some children carry over lack of understanding from the written algorithm. Here is an explanation of 57 – 29:
  
  '9 - 7 = 2, 5 - 2 = 3, so answer is 32.'

- One teacher gave the year 2 class a number story ('I had 17 apples; I cooked 7. How many do I have left?'). The children wrote down the story and worked out an answer. The teacher then asked some children to explain their strategy. Some of the responses were:
  
  'If you take away the 7, it only leaves the ten.'

  '10 + 7 is 17, so 17 - 7 is 10.'

  'I counted backwards and got 9.'

- Another story was: 'Four men walking down the road. How many legs.'
  
  Two responses to this were:

  'Two people have four legs, then another two have four legs, four and four is eight.'

  'I looked under the table and counted legs.'

- Some responses to the story 'If I had 20 balls and shared them among five children, how many balls each? were:

  'I got 4 and 10. Then I put crosses and worked it out.'

  'I went 4, 8, 12, 16, 20.'

  'I counted round my fingers, that was 5. Then round again and again and again. Four times round made twenty.'

**FIND MY NUMBER**

**Objective:** to develop confidence with numbers and use a variety of strategies for mental computation

**Materials:** blackboard

**Organisation:** whole class or groups of 2–4

**Procedure:**

1. Teacher mentally selects a number between 1 and 50, and writes it down without revealing it to the children.

2. Children or groups take turns to ask questions to which the teacher only answers 'yes' or 'no'.

3. Teacher writes each question and answer briefly on the board, for example:

<table>
<thead>
<tr>
<th>More than 30?</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even?</td>
<td>Yes</td>
</tr>
<tr>
<td>Ends in 6?</td>
<td>No</td>
</tr>
<tr>
<td>Multiple of 3?</td>
<td>No</td>
</tr>
</tbody>
</table>

4. When 3 or 4 questions have been asked, teacher asks, 'What is the smallest number it still could be? What is the largest? What else could it be?'

Children discuss this in groups. Teacher records possible numbers on the board:

2 4 8 10 14 20 22 28

5. Game continues. As each question is asked, children tell teacher which number(s) can now be crossed out and eliminated.

6. Keep record of how many questions are asked before number is found.

7. When children are familiar with the game, discuss strategies. How many questions might be needed? Is 'yes' a better answer to get than 'no'?
Comments on the activity:
• Write the questions in any convenient short form as long as the meaning is clear to everyone. For example: ‘M3’ is much easier than a longer ‘Is it a multiple of 3?’
• Teachers noted an immediate improvement in the ability of children to look at numbers and ask questions about them.
• Teachers commented on the non-threatening nature of the activity. When children ask a question they are not threatened by the answer. It is giving information to them.
• A year 2 teacher commented: ‘I’ve been surprised by the concepts and skills the children are showing in this activity. I don’t know whether they had it in them and I didn’t realise, or whether this activity is developing them.’ She also mentioned that the activity helped the children’s language skills.
• A year 1/2 teacher draws a 0 to 20 number line on the board because she believes it helps the children to visualise the numbers.
• It is important that children ask questions that are clear. One child asked: ‘Does the number have a 4 in it?’ The teacher interpreted this as ‘Is the tens and/or the ones column 4?’ The discussion that followed revealed that the child meant: ‘Is it a multiple of 4?’
• Model questions for the children. For example ‘Is it odd or even?’ is not a yes/no question. ‘Is it even?’ is correct.
• It is vital that at some point during the activity the teacher stops the activity and reviews the effect of the questions so far, so that all children can get back into the game. One teacher does this after 3 questions. Another stops after 3 questions and asks children for a sentence which sums up the effect of these questions: ‘It is an odd number, less than 30, and not divisible by 7.’
• In older classes, children can come to the front and answer the questions. However some difficulties have arisen:
Q  Is it a square number?
A  What’s a square number?
In addition children have been observed not hearing, or not accepting, questions from some children, and answering ‘No’ in error (this is very frustrating for the others). Close monitoring by the teacher can overcome these difficulties.

Some ways teachers have adapted this format:
• A year 1 teacher writes all the numbers on the board first (they were using numbers 1 to 10) then asks each question she asks: ‘Which numbers can we cross off now?’
• A year 6 teacher has the children play the game in groups of 4, to promote discussion.
• A year 1 teacher who was introducing this activity for the first time wrote on the board a list of questions from which the children could choose:
  Is it more/less than . . . ?
  Is it an odd/even number . . . ?
  Is it the number before/after . . . ?
  Is it between . . . and . . . ?
  She kept these questions on the board for the first few sessions until the children were familiar with them. Later she encouraged the children to ask other questions as well.
• A year 6 teacher as well as writing the questions and answers on the board, also asks the children for the effects of each answer, and writes these on the board. For example:

<table>
<thead>
<tr>
<th>Questions</th>
<th>Answers</th>
<th>We know that it is . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odd?</td>
<td>No</td>
<td>even</td>
</tr>
<tr>
<td>Multiple of 7?</td>
<td>No</td>
<td>not 7, 14, 21, 28, 35, 42, 49</td>
</tr>
<tr>
<td>In the 20s?</td>
<td>No</td>
<td>in the ones, 10s, 20s, or 40s.</td>
</tr>
<tr>
<td>In the 30s?</td>
<td>Yes</td>
<td>32, 34, 36 or 38</td>
</tr>
<tr>
<td>Over 35?</td>
<td>Yes</td>
<td>36 or 38</td>
</tr>
<tr>
<td>36?</td>
<td>Yes</td>
<td>38</td>
</tr>
</tbody>
</table>

A year 7 teacher, instead of writing the questions, writes the answers: It doesn’t have an 8
Yes, it has a 6
• A year 6 teacher does not allow ‘between’ questions (‘Is it between 20 and 50?’) because she wants children to think of other aspects of numbers.
• A year 2 teacher on the other hand only allows ‘Is it smaller than . . . ? Is it larger than . . . ?’ questions because she is emphasising ordering numbers at present.
Another year 2 teacher said that her class uses the words 'Is it before/after . . . ?' because they find 'bigger than/smaller than' difficult at present. A year 1 teacher uses only 'higher' and 'lower'.

A teacher has three children sitting in front of the classroom facing the other children. The teacher writes a number on the board. The three children, who cannot see the number, ask questions of the rest of the class.

Extracts from lessons:
- **Is it a tidy number (i.e. ends in zero)?**
  The children invented the use of 'tidy' because they found 'compatible' a less friendly word and also difficult to say.
- **Does it (i.e. the digits) add up to 5?**
  At first children ask very straightforward questions, but here a child has seen the number in a different way.
- **Is it odd?** Yes
- **Is it divisible by 4?**
  Here is a clear example of a child not understanding, or not taking account of, a previous question.

**COMPATIBLE NUMBERS**

**Objective:** to recognise that mental calculations can be simplified through use of this 'compatible numbers' strategy.

**Materials:** blackboard or overhead projector

**Organisation:** whole class.

**Procedure:**
1. Write approximately twenty compatible numbers (numbers that when added produce a tidy sum—usually one ending in a zero) in random fashion within a frame, e.g. in junior grades, use compatible numbers that total 10; in middle and upper grades, use compatible numbers that total 100.

<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>9</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>60</th>
<th>56</th>
<th>88</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>71</td>
<td>33</td>
<td>40</td>
</tr>
<tr>
<td>12</td>
<td>96</td>
<td>4</td>
<td>29</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>70</td>
<td>75</td>
</tr>
</tbody>
</table>

2. Ask children to list pairs of compatible numbers.
Comments on the activity:

- When you add compatible numbers, a tidy sum is produced (that is, one usually ending in a zero).
e.g. 7 + 3  16 + 24  38 + 62  1.72 + 0.18
- The reason for concentrating on sums of two numbers ending in a zero is that most people use these sums to simplify mental computations. For example, when adding 89 and 36, many children and adults will say ‘89 and 11 makes 100, and 25 more makes 125.’
  You cannot use this strategy efficiently unless, on seeing 89, you can immediately calculate that 11 more is needed to make 100.
- It is important that children write down the pairs they find, and do not simply cross them off. If they do the latter, they end up with every number crossed out and no indication whether the numbers were correctly paired.
- It is important that some discussion follows the activity:
  How did you work that out?
  What was your strategy?
  Why did you use that strategy?

Some ways teachers have adapted this format:

- One teacher asks her class to write as many compatible pairs as they can in one minute.
- One teacher gives frames to children to work in pairs.
- A year 7 teacher invites the children to make up their own grids and swap them with their partners. Sometimes he lays down conditions:
  Use only numbers ending in 5 or 0.
  Only use odd numbers.
  Don’t use numbers ending in 5 or 0.
  For a change, one teacher asks for 3 numbers which make a ‘tidy’ sum.

Extracts from lessons:

- A year 5 teacher wrote these numbers randomly on the board:

<table>
<thead>
<tr>
<th>39</th>
<th>41</th>
<th>47</th>
<th>68</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>92</td>
<td>53</td>
<td>87</td>
<td>8</td>
<td>59</td>
</tr>
</tbody>
</table>

  The children were given two minutes to find and record the pairs. The number of pairs found ranged from 2 to 7.

- A year 1 teacher had the numbers 0 to 10, with a second 5, written in a 4 by 3 grid on the board. As the children spotted pairs she wrote them on the board. The ones was put on the whole class to spot any errors. For example one child gave 3 + 9. The teacher wrote it on the board since no-one challenged it. When the list of pairs was complete the teacher went through each answer and asked ‘Does it equal 10?’ The children responded ‘It’s too much!’ They discussed, using counters, why it was too much.
SUM STORIES

Objective: to establish the vital link between number symbols and the child's world

Materials: blackboard

Organisation: whole class

Procedure:
1. The teacher writes a calculation on the board, e.g. 15 – 6 = ? Note that the calculation is written horizontally.

2. The children offer a variety of real-life situations in which the calculation could occur: 'There were fifteen biscuits in the packet but we ate six so there were only nine left.' Establish the answer before asking for stories.

3. Children are often slow to offer stories at first but produce them much more freely once the idea is clear. Encourage creativity and variety in the situations. Stories often revolve around money and food; limit the number of stories you will accept on the same theme.

4. It is usually best to keep the size of the number reasonably simple, but children should be able to attach a meaning to any calculation which they are performing.

Comments on the activity:

- This activity is important because, while it does not involve mental calculation, it does help to establish that vital link between number symbols and the child's world.

- A year 1 teacher commented that talking about mathematics in this 'story-telling' way before children write seems to help their writing, so it is tying in well with the language program.

- The child's world of course includes the world of fantasy; for 72 – 29 one child offered 'The dragon had 72 teeth but the giant knocked 29 of them out.' In a way this is more 'real' than the child who, for 7 x 50 offered 'The house had 7 rooms and 50 people lived in each room.'

- When the computation is a subtraction, you may find that almost every child offers a 'take-away' story (one favourite of ours is '10 teachers at a meeting, 5 were talking, 5 were not.') while almost nobody offers a comparison ('I have 10 dollars, my brother has 5 dollars, I have 5 dollars more than him').

- It is common for children to offer stories about a limited range of subjects: themselves, money and food. When this happens, you can widen the range by saying:

  No more money stories.
  Who can give me a story about . . .

- It is important to focus on the calculation (for example 8 x 2) not only on the answer (16). The stories the children give should represent 8 x 2, not just 16, and not 2 x 8. 'A boy went to school with 8 marbles and won another 8' is not correct for 8 x 2.

Some ways teachers have adapted this format:

- A year 6 teacher used the activity to check on the understanding of zero and of decimals:

  15 – 0 =
  There were 15 parked cars. Thieves couldn't break into any of them. There were 15 left.

  4 + 1.5 =
  Four litres of orange juice in a jug. I added 1.5 litres. We had 5.5 litres.

The teacher was surprised to find that no other measurements or money stories were offered. It gave her a good insight into the limits of the children's understanding of use of decimals in real life.
• A year 3 teacher finds that the children work well in pairs on this activity, discussing and evaluating each other's stories.

• A year 7 teacher gives children a number (e.g. 28) and they then write calculations and calculation stories about it in a special book.

A square has 7 cm sides. What is the perimeter?

56 people at a party divided into 2 groups. There were 28 in each group.

56 people spent 50c each on a birthday present. How many dollars did they have to spend?

If I got $3.50 a week, how much did I have after 8 weeks?

A pentagon has sides of 7 cm, 6 cm, 8 cm, 3.5 cm and 3.5 cm. What is its perimeter?

A dog has 4 legs. Multiply its legs by 7 and you get 28. (This clearly shows an inability to translate the symbols into a practical situation.)

• For $47 \times 5 + 78$, one girl wrote 'At a party 47 children each brought 5 balloons making a total of 235. But the home in which the party was being held already had 78 balloons. Altogether there were 313 balloons.'

Extracts from lessons:

• For 16 - 7 one year 4 child offered 'There was a teenager aged 16. He went in a time machine back 7 years. He was now 9 years old.'

• For 4 + 2 = 6, a year 1 teacher was surprised and delighted when one child said: '4 apples and 2 bananas make ... (long pause) ... 6 pieces of fruit!'

• For 21 + 9 a year 4 child wrote: 'It was the 21st. 9 days later it was the 30th.'

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**TEACH A STRATEGY**

**Objective:** to use a variety of strategies for mental computation

**Materials:** blackboard or overhead projector

**Organisation:** whole class, then pairs

**Procedure:**

1. Give children a problem that lends itself to solution by the strategy that is being emphasised (see pages 41–2 for examples of suitable strategies). For example, when introducing the 'work from the left' strategy the following problem could be used: $28 + 37$.

2. Ask children to solve the problem mentally.

3. Ask some children to explain their methods for solving the problem.

4. Focus on children using the strategy being emphasised. Be careful not to discourage alternative methods but highlight the strategy under consideration, e.g. $28 + 37$

   $20 + 30 = 50$

   $8 + 7 = 15$

   $50 + 15 = 65$

5. Blackboard other examples to give children practice in using this strategy.

6. The children can explain their methods of solution to their partners.
Comments on the activity:

- Research shows that children use a variety of strategies to perform mental calculations. This format can be used to provide instruction in some of these strategies. An explanation of the more common strategies is given on the following pages.
- Some teachers found that this activity was more directed and enjoyed less by the children. However, it is worth taking some of the more commonly used strategies and letting the children practise that particular strategy on a few calculations. A good time to do this might be if children themselves have produced a useful strategy.
- There should never be any suggestion that the strategy being practised is the only one, or the best one, to use for these particular calculations. The hope is that some children will find the strategy ‘friendly’ and incorporate it into their tool-kit of strategies for future occasions.

Some ways teachers have adapted this format

- One year 6 teacher colours the ones blue and the tens red to prevent any ambiguity when describing a strategy. ‘Which did you add first, the blue numbers or the red ones?’
- A year 1 teacher found that children enjoyed and benefited from working in pairs on this activity.

Common Strategies

Pages 41–2 contain a list of the strategies most commonly used by children, with an example of the steps for each strategy.

For example, the first strategy (Work from the Left—Addition) involves first adding together the tens (the left hand digits) then adding together the ones while holding the first sum in memory, and finally adding these two sums.

1 Work from the left (Addition)
28 + 37 = ?
20 + 30 = 50
8 + 7 = 15
50 + 15 = 65

2 Changing division to multiplication
12 ÷ 3 = ?
3 × ? = 12
3 × 4 = 12

3 Changing subtraction to addition
14 – 9 = ?
9 + ? = 14
9 + 5 = 14

4 Commutative property of addition
3 + 9 = ?
That’s the same as 9 + 3

5 Commutative property of multiplication
4 × 7 = ?
That’s the same as 7 × 4

6 Doubling and halving
2 × 8 = ?
That’s 8 + 8
18 × 15 = ?
Half of 18 is 9 Double 15 is 30
That’s the same as 9 × 30 or 270

7 Near Doubles
6 + 7 = ?
That’s 6 + 6 + 1 or 7 + 7 – 1
8 Add or subtract parts
37 + 24 = ?
37 + 20 = 57
57 + 4 = 61
94 - 26 = ?
94 - 20 = 74
74 - 6 = 68

9 Bridging Tens and Hundreds
9 + 4 = ?
9 + 1 = 10
10 + 3 = 13
26 + 7 = ?
26 + 4 = 30
30 + 3 = 33
74 - 26 = ?
74 - 4 = 70
70 - 22 = 48

89 + 17 = ?
89 + 11 = 100
100 + 6 = 106

112 - 34 = ?
112 - 12 = 100
100 - 22 = 78

WITHIN LIMITS

Objective: to develop estimation skills
Materials: blackboard and objects to estimate, if needed

Procedure:
1. Arrange the children in groups of from 2 to 4.
2. The teacher chooses something for the class to estimate (for suggestions see next page.)
3. Each group has to discuss and agree on a higher and a lower limit within which they are certain that the actual measurement lies.
4. The teacher writes headings 'greater (more) than' and 'less (fewer) than' and enters an extreme value for each as the first entry in each column, for example, if estimating the number of marbles in a jar, the teacher can write (more than) '1' and (fewer than) '1000' as starting points.
5. Each group is invited to narrow either of these limits: the teacher will only write a number on the board if all other groups agree.
6. When no further narrowing of the limits can be agreed on, a midpoint between these limits can be calculated and compared with the actual answer.

Here is an example in which the final agreed limits are 230 and 450. The numbers crossed out at the foot of each column are limits which were offered by one group but not accepted by at least one other group. They are written on the board so as to remind the groups that there is no point in going within these bounds.

<table>
<thead>
<tr>
<th>How many marbles in the jar?</th>
<th>More than</th>
<th>Less than</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>600</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>480</td>
<td></td>
</tr>
<tr>
<td>230</td>
<td>450</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>400</td>
<td></td>
</tr>
</tbody>
</table>
Comments on the activity:
- Almost anything can form the basis of an estimate. Some examples are:
  - length of classroom, desk, book
  - height of classroom, table, tree
  - area of blackboard, classroom floor, piece of paper
  - capacity of matchbox, jar, bucket, room
  - number of beans in a jar, people in a photograph, pages in a book
  - value of a pile of money, a mixed pile of MAB pieces
  - number of seconds in a given interval of time
  - the answer to a difficult calculation
- This activity provides an enjoyable alternative to the idea that an estimate has to be 'roughly right', by substituting the equally mathematical idea that an estimate sets specific bounds within which you are sure that the answer lies.
- Teachers have noticed how often the 'average' of the two limits is a good estimate. Children could be encouraged to use this strategy to help them arrive at an estimate.
- An important part of the activity is the discussion within groups as they try to agree on the limits. Trying to convince others by reasoning: describing the strategies used; and negotiating an agreed decision are valuable features.
- The teacher can gain valuable information about areas of weakness in practical measurement and children's ideas of metric and other units, particularly when the actual measurement is seen to fall far wide of their agreed limits.

Some ways teachers have adapted this format:
- One teacher has the children decide on the limits individually, then has a class discussion about the range of results and the strategies used.
- Another teacher has the children discuss and agree in pairs, then two pairs discuss and agree, and so on.
- One teacher uses the same object for a week, asking for different estimates each day. For example, she brings a jar of beans and asks, on successive days:
  - How many beans in the jar?
  - How high is the jar?
  - If you laid all the beans in a row, how far would the row be?
  - What is the capacity of the jar?
  - How many jars would you need placed end to end to reach across the classroom?
- A teacher in a year 6 class invited the children to make up their own estimation questions—with answers!
- Instead of using the two columns, one teacher draws a number line on the board and marks the successive limits accepted. She finds this helps her class to picture the limits closing in:

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0 150 250 450 500 600 1000
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- Some teachers of younger children encourage them to use measuring implements, or pace up and down the room to help them estimate. Older classes are expected to estimate from their desks.
- Assessment can be done by giving children measures or amounts to estimate before and after some weeks of estimating. Much can also be learned by listening to a child explaining a strategy used or the reasons for an estimate given.